

A history of theory of structures in the nineteenth century

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1

Introduction

Much progress in theory of structures during the nineteenth century has been ascribed, notably by Clapeyron (1857) in France and Pole (Jeaffreson, 1864) in Britain, to the coming of the railway era. But the state of knowledge of the subject at the beginning of the century was ripe for rapid development due, for example, to Coulomb's remarkable research in applied mechanics. Early in the century Navier began to contribute to engineering science encouraged by his uncle, M. Gauthey, Inspector General of bridges and highways in France. Navier was born in 1785 and orphaned when he was fourteen years of age. He was adopted by Gauthey whose book on bridges he published (1809) and revised (1832), following his education at L'Ecole Polytechnique and then at L'Ecole des Ponts et Chaussées, from where he had become ingénieur ordinaire in 1808. He may be regarded as the founder of modern theory of elasticity and its application to structures and their elements. The year 1826 is memorable for the publication of Navier's celebrated *Leçons* as well as for the completion of Telford's remarkable wrought iron chain suspension bridge at Menai (it was also a year of sadness for Navier due to failure, prematurely, of the Pont des Invalides, a Paris suspension bridge which he had designed).

There is little doubt that France then led the world in the application of scientific principles to practical problems, having established, in 1784 and 1747 respectively, those two outstanding places of learning attended by Navier (l'Ecole Polytechnique was at first directed by Monge who was later joined by such outstanding figures as Fourier, Lagrange and Poisson: l'Ecole des Ponts et Chaussées was at first under the direction of Perronet who was a distinguished bridge engineer; later, Prony was its Director).

It is tempting to assert that the needs of engineering practice generated

by, say, new materials, led to relevant advances in applied mechanics. Some major advances were, however, premature, apparently stimulated, at least in part, by the natural curiosity of individuals. Unfortunately, such premature advances seem to have received little attention and, being overlooked by practitioners, were forgotten, only to be rediscovered much later (and sometimes applied in an inferior manner), for example Navier's theory of statically-indeterminate bar systems.

Initially, masonry, timber and cast iron were the principal materials of construction, and their properties dictated the nature of structural forms: the arch to utilise the compressive strength of masonry or cast iron; the beam and latticework to utilise both the tensile and compressive strengths of timber. The manufacture and rolling of wrought iron (strong in tension and compression) was in its infancy in 1800 but it was to have a profound effect upon the theory and practice of construction.

In Britain the influence of the Rev. Professor Henry Moseley F.R.S. and his disciple Pole (later, professor at University College, London) on those renowned pioneers of railway construction, Robert Stephenson and Isambard Kingdom Brunel, is especially illuminating. (It shows, moreover, that contrary to widespread belief and Culmann's critical remarks, noted below, early outstanding advances in iron bridge construction in Britain were the result of using advanced scientific principles and experimental techniques as well as ingenuity of construction.) Moseley was noted for various contributions to engineering science and was familiar with French engineering science, especially the work of Coulomb, Navier and Poncelet. He was a pioneer of engineering education along with others including Robison, Willis and Rankine; but their influence does not seem to have been sufficient to achieve, in Britain, an enduring unity of theory and practice (the subject, incidentally, of Rankine's Inaugural Lecture to the University of Glasgow in 1856) such as that which was typified in the Britannia Bridge.

Moseley received his education in France as well as in Britain. He graduated in mathematics at St John's College, Cambridge and in 1831, when thirty years of age, he became professor of natural philosophy and astronomy at King's College, London, where he carried out research in applied mechanics, a subject which he taught to students of engineering and architecture 1840–2. In 1843 (the year before he left London to become, first a Government Inspector of Schools, and then, in 1853, a residentiary canon of Bristol Cathedral), his book *The mechanical principles of engineering and architecture*, which was based on his lectures, was published. It is probably the first comprehensive treatise on what might

be called modern engineering mechanics to appear in English. There are acknowledgements to Coulomb (especially with regard to arches, earth pressure and friction), Poncelet (notably on elastic energy) and Navier (deflexion of beams and analysis of encastré and continuous beams). The appearance throughout of principles of optimisation (*extremum* principles relating to mechanical devices as well as statics) is noteworthy.

Moseley devoted much effort to a rigorous analysis of the stability of masonry voussoir arches (1835), from which emerged the concepts of line of pressure and line of resistance and the ‘principle of least pressures’. (See Chapter 3: that principle has not survived but it inspired fruitful research by Cotterill.) But though his work on arches seems to explain observed modes of failure, it was to cause more confusion than enlightenment among British engineers, in spite of the interpretive efforts of Barlow (1846) and Snell (1846). Heyman has noted (1966) that the theory of the stability of a masonry arch due to Coulomb (1776) is unsurpassed. It is interesting, though, that Moseley concluded by vindicating the designs of arches by Rennie and others, based on the so-called ‘wedge theory’ of simple statics neglecting friction. The latter has been variously ascribed to Hooke, De La Hire, Parent and David Gregory (Charlton, 1976*b*). Indeed, that elementary theory was used throughout the century, notably by Brunel, without adverse consequences and independently of the ultimate strength theories of Coulomb and Villarceau.

It is for the design of continuous tubular wrought iron plate girder bridges (among the earliest continuous iron bridges in Europe) that Moseley’s work is especially significant. In 1849 the theory of continuous beams, due to Moseley’s teaching of Navier’s methods, was used both for the Britannia Bridge and for the Torksey Bridge of John Fowler. This latter structure was continuous over three supports and, although it was very much smaller and altogether less enterprising than the Britannia, it was, nevertheless, of much interest (as noted in Chapter 2). It was completed nearly two years before the Britannia Bridge and declared unsafe for public use by the Government Inspector, Captain Simmons, R.E., in the atmosphere of suspicion of iron railway bridges which followed the Royal Commission of 1847–8 to inquire into the application of iron (railway) structures ‘exposed to violent concussions and vibration’ (Stokes, 1849). (That was after the fatal accident at Chester, for which Simmons was Inspector, when a trussed cast iron girder railway bridge by Stephenson collapsed while carrying a train.) The matter came to a head when Fairbairn, patentee of the tubular girder (1846), according to Pole (Jeaffreson, 1864), read a paper ‘On tubular girder bridges’ before the

Institution of Civil Engineers (1850). It was followed by a lively discussion which concentrated on the safety of the Torksey Bridge. Pole, Captain Simmons, Wild and Professor Willis were among the leading participants. Pole described his detailed analysis of the structure (Chapter 2), along with his results which supported the calculations and experiments of Wild for Fowler and vindicated the safety of the bridge. The Secretary of the Institution, Manby, strongly criticised government interference as being detrimental to progress. The contribution of Willis, Jacksonian Professor of Natural Philosophy at Cambridge, was concerned with the dynamic aspect especially. He was a member of the Royal Commission and distinguished for his experiments on the effect of a load travelling across a metal beam (1849). (His collaboration with Professor Stokes resulted in the latter's celebrated paper of 1849 and heralded the beginning of the precise study of dynamics of structures (Chapter 11), a subject which attracted Navier's attention with regard to suspension bridges.)

On the continent of Europe, theory of continuous beams (Chapter 2) was pursued, after Navier, notably by Clapeyron in 1848. Clapeyron, with Lamé, became professor at The Institute of the Engineers of Ways of Communication in St Petersburg in 1823 and, according to Timoshenko (1953), had a profound influence on the development of theory of structures in Russia. (Jourawski of that institute was engineer for the first railway bridges in Russia and developed theory of trusses as long ago as 1847, as well as exploiting continuous beams.) Among their early works was a memoir on the analysis of arches (1823), and Chalmers (1881) notes their work in graphical statics (1826*b*). Clapeyron is remembered popularly for his theorem of the three moments for continuous beams (1857), which resulted from the construction of the Pont d'Asnières near Paris. The idea was published first, though, by Bertot in 1855. Among the other contributors to aspects of continuous beam theory were (Chapter 2) Bresse, Mohr and Winkler; and it is important to acknowledge the method due to the elastician Clebsch, given in his celebrated book published in 1862 (and again, in 1883, in French, with annotations by Saint-Venant with Flamant). Subsequently, graphical methods (due to Mohr and especially Claxton Fidler) alleviated continuous beam analysis for some engineers.

It is interesting that Moseley and others seemed to overlook Navier's elegant method of analysing statically-indeterminate systems of bars (pin-jointed systems), a subject which was to be approached anew by Maxwell (1864*b*) and Mohr (1874*a*) as the need arose. Also, Clebsch included it in his book and developed it further. He treated space systems and introduced stiffness coefficients of linear elasticity with their reciprocal

property (so, in a sense, establishing priority for the reciprocal theorem over Maxwell, Betti and Rayleigh, but he did not deal, it seems, with the physical implication). Another eminent elastician aware of the whole of Navier's *Leçons* was, of course, Saint-Venant who was responsible for the third (1864) edition. It seems that leadership in theory of elasticity and structures began to pass from France to Germany soon after that.

Navier had also provided the basis for elastic arch theory in his *Leçons* where the solution of the two-pin (statically-indeterminate) arch appears, for example. Bresse (1854) is, however, usually credited for this aspect having regard to his extensive treatment. It was at this time that Bresse noted and exploited the principle of superposition for linearly elastic structures (and linear systems generally) with regard to symmetry and anti-symmetry (Chapter 3). Culmann, Mohr, Winkler and Müller-Breslau were among later contributors to arch analysis and, according to W. Ritter (1907), it is to Culmann that the concept of the elastic centre is due, though Mohr seemed to identify the device (Chapter 10) with respect to a framed arch. Winkler's theory of stress in elements of large curvature (such as crane hooks) is noteworthy and he and Mohr c. 1870 suggested the use of theory of elasticity for analysing stone or masonry arches.

The economies apparently afforded by lattice girders and trusses in relation to arch and plate iron structures were being explored in the middle of the century, by which time mass production of wrought iron sections had begun. Those economies appeared to be related to the increase in the length of single, simply supported spans, which was afforded by the reduction in self-weight per unit length of such construction (in comparison with conventional beams) and to the inherent convenience of the beam as such. The flat-strip lattice girder was first known in Britain in the form of Smart's patent iron bridge, after Smart who, according to O. Gregory (1825), invented it in 1824. Its design was apparently based on the beam theory, the latticework being assumed equivalent to a plate web. These forms of construction were developed enthusiastically in the U.S.A., using timber as well as iron to build some impressive viaducts. Whipple achieved fame there for his truss designs based on sound principles, and his book published in 1847 contains, it is believed, one of the earliest thorough treatments of the analytical statics of complex frameworks (without redundant elements). The Russian Jourawski is said by Timoshenko (1950, 1953) to have initiated precise analysis of trusses, while Emmerson (1972) believes that Robison (professor of natural philosophy at Edinburgh) did so some fifty years earlier for timber trusses. Robison is also acknowledged by Cotterill (1884) and Weyrauch (1887) but Straub (1952) dismisses him

as unsound. Whipple attempted to achieve true pin-jointing of his trusses, whereas the gusset plate and rivetting was used elsewhere, for example in Britain.

Both Moseley and Weale omitted the truss from their books (1843), indicating, apparently, that it was not then being used on a significant scale. Indeed, the theoretical content of Weale's *Theory, practice and architecture of bridges* (1843) deals mainly with masonry arches and suspension bridges and was contributed by Moseley and his colleague at King's College, London, W. Hann.

Although, in Britain, lattice girder railway bridges had appeared by 1844 (Hemans, 1844) the earliest truss for bridges seems to have been due to Captain Warren. According to Pole (Jeaffreson, 1864) it was used first for a major bridge (London Bridge Station) in 1850. Then in 1852 Cubitt used it for carrying a branch line of the Great Northern Railway over the Trent near Newark. The Warren–Kennard girder (de Maré, 1954) was used in the building of the spectacular Crumlin Viaduct (Ebbw Vale, Monmouthshire) in 1857 and also in the Melton Viaduct (Okehampton). At about that time Bouch, assisted by Bow, was using the now so-called 'double' Warren girder for viaducts of the South Durham and Lancashire Union Railway, including the impressive Belah Viaduct. Indeed, Bow notes (1873) that these projects began in 1855. The viaducts consisted of a number of simply supported spans, and elementary statics was used for their design. Whewell's *Mechanics applied to the arts* (1834) and *The mechanics of engineering* (1841) exemplify the sound knowledge of elementary analytical statics available to engineers. (In the latter work, dedicated to Willis, the principle of using elastic properties to deal with statical indeterminacy is described briefly.)

The truss and its design provided impetus for the development of graphical methods of analysis. Bow (1873) recalls seeing a paper by Wild in 1854, which gave a complete graphical analysis of a simple truss (Bow, 1873, Fig. 243(i)), though he believed the date of the paper to be earlier. Graphical methods brought truss analysis within the competence of engineering draughtsmen and their origin is usually ascribed to Rankine (1858) and Maxwell (1864a), notwithstanding Bow's acknowledgement of Wild. But the use of graphical analysis in statics was not new. Varignon's funicular polygon (1725) and Coulomb's celebrated work in respect of earth pressures and masonry structures (Heyman, 1972) are noteworthy examples of the use of graphical analysis in the eighteenth century. Early in the nineteenth century, while he was a prisoner of war, Poncelet's interest in geometry for analytical purposes resulted in the creation of the

new (projective) geometry (c. 1813), according to Chalmers (1881). Poncelet undoubtedly came to be regarded as a creator of modern engineering mechanics. Later, the initiative for extensive application of graphical methods was taken by Culmann at Zurich.

Culmann became professor of engineering sciences at the Zurich Polytechnikum in 1855 after experience in railway construction, and at a time of heavy demand on knowledge of theory of structures for the design of novel economical bridges on the truss principle. His highly distinguished contributions to graphical analysis, culminating in his celebrated book published in 1866, established his dominance of the new discipline called graphical statics which embraced engineering analysis generally. He based much of his work on the 'new geometry' of Poncelet (and Möbius). Curiously, he denied credit to Maxwell for the concept of reciprocal figures in framework analysis but Cremona (professor of mathematics at Milan and later at Rome), to whom he gave credit for it, acknowledged Maxwell's priority in a particularly lucid account of the subject (1872; see also Chapter 4).

Culmann's rejection of Maxwell probably reflected his disdain for British engineering (like Navier he visited Britain and the U.S.A., though nearly thirty years later, to study advances in bridge engineering which was at that time mainly related to railways). His opinions in this respect are recorded by Chalmers (Chapter 4), an ardent admirer of Culmann, who gives an extensive historical review in the preface of his scholarly book (1881). He quotes Culmann (from his book of 1866): 'But what is appropriate to the rich Englishman, who everywhere carries himself about with great consciousness, "I am in possession of the iron and do not require to trouble myself about statics", is not so to the poor devils of the Continent....' He contrasted the differences between the Continent and Britain with regard to the preparedness for Culmann's powerful methods, referring to the University and High School system in Germany where students were familiar with the works of Poncelet, Möbius and Chasles; while in Britain, the 'modern geometry' received little attention. Chalmers deplored the failure in Britain to accept the vital need for scientific training of engineers, thus:

There are, no doubt, among us, a large number who in earlier years have studied their Pratt, their Navier, their Moseley, or who in more recent years have become familiar with their Bresse and Rankine, have made themselves familiar with Clapeyron's Theorem of the Three Moments, even a few to whom Lamé is not unknown but those have done so without hope of reward.

Moreover, Weyrauch is quoted as asserting (1873) 'that continuous

beams are popular only in countries where engineers can calculate' (a criticism of Britain which might well have been valid in 1880 but which neglected the priority attaching to the Britannia Bridge in both science and technology some thirty years earlier).

In 1881, Culmann died and was succeeded by Wilhelm Ritter, his former pupil and professor at the Riga Polytechnic Institute. Ritter published a major work on graphical statics (1888–1907). But his work should not be confused with that of August Ritter, Culmann's contemporary at Aix-la-Chapelle, whose book on theory and calculation of iron bridges and roofs (1862) was translated into English by Captain Sankey. In the course of that task, Sankey observed that Ritter's so-called method of moments (or sections) had already been discovered by Rankine (1858) when Ritter's book first appeared. (Schwedler is sometimes credited with the same idea.)

A distinguished contemporary of Culmann was Levy, in France. A pupil of Saint-Venant, he made a profound contribution to the modernisation of graphical statics in France, with his book, published in 1874, which contained *inter alia* some original matter of a purely analytical nature with regard to structures (Chapter 6). Williot's method of finding deflexions of trusses graphically (1877) is also a significant French contribution.

In Britain the degree of sophistication in graphical statics was less than on the Continent, as noted by Chalmers. Nevertheless, in addition to the contributions of Maxwell (1864*a*), Jenkin (1869), and Bow (1873) who was famed for his notation for force diagrams of trusses, there were the distinguished contributions of Claxton Fidler's analysis by 'characteristic points' of continuous beams (1883) and Fuller's graphical method for arches (1874). (Moreover, Maxwell was, after Jenkin's encouragement in 1861, also concerned simultaneously with the analysis of statically-indeterminate frameworks.)

On the Continent, Castigliano in Italy (1873), Levy in France (1874) and Mohr in Germany (1874*a*) had achieved priority in various ways of analysing statically-indeterminate trusses or bar frameworks, in addition to devoting much attention to graphical analysis where appropriate. Levy, mindful of Navier's method published in his *Leçons* some fifty years earlier, suggested an alternative approach, while Mohr appears to have attacked the problem *de novo* in a manner essentially the same as that of Maxwell (the details of these methods are described in Chapters 5 and 6). Castigliano, with full knowledge of Navier's method (like Levy) sought (as described in Chapter 8) an alternative based on elastic energy derivations (after Menebrea's unsuccessful attempt of 1858). Castigliano gives, incidentally, an account of the method due to Navier in his book published in 1879

(it seems that Navier's *Leçons* were translated into Italian). Incidentally, Castigliano, like so many other leaders of structural analysis in Europe (Clapeyron, Culmann, Jourawski, Engesser, Mohr, Rankine, Jasinsky, Winkler and Crotti), was concerned with railway construction.

Having (like Mohr and Levy, after Culmann) developed sophisticated graphical methods for dealing with a wide variety of problems (of which truss analysis was only one) Müller-Breslau turned his attention eagerly to promoting and extending the new analytical methods of Maxwell–Mohr and Castigliano. (Mohr's very different attitude to both Castigliano and Maxwell is described in Chapter 10.) Müller-Breslau's major work on graphical statics (1887*b*) includes much that is not concerned with graphical or geometrical methods (after the manner of Levy) and adds a high degree of sophistication and clarity to the analysis of statically-indeterminate frameworks. Thus, he appears to have introduced uniformity by the notation of flexibility (influence coefficients) for linearly elastic structures and so to have relieved the formulation of the solution, for any type of structure, from any particular method of calculating deflexions (that is, the flexibility coefficients would be calculated in a manner which depended upon individual preference). Müller-Breslau is popularly remembered for the theorem regarding influence lines for forces in elements of statically-indeterminate structures, which bears his name (Chapter 10). He is also credited (as is Southwell) with the concept of tension coefficients for space frame analysis, but the concept is implied by Weyrauch (1884).

In the meantime Engesser, and Castigliano's friend Crotti were adding to knowledge of energy principles in theory of structures. (Castigliano's theorems of strain energy, especially his so-called 'principle of least work', for relieving analysis of statically-indeterminate structures of conceptual, physical thought, quickly received widespread acclaim on the Continent and in Britain later, due to Martin (1895) and Andrews (1919).) The origins of energy methods in theory of structures or practical mechanics may be traced to Poncelet (Chapter 7) and Moseley. (Part 2 of Castigliano's first theorem of strain energy was anticipated by Moseley (1843).) Indeed, Moseley's disciple, the mathematician Cotterill, anticipated the essence of Castigliano's principle of least work in 1865 in the *Philosophical Magazine*, which escaped the notice of engineering scientists in Europe and, later, Fränkel (1882) discovered it independently. Apart from their philosophical interest, involving speculation regarding economy of Nature, the *extremum* principles resulting from research into energy concepts were to be valuable, mainly for obtaining rapid approximate solutions to certain complicated problems.

Castigliano's least work theorem was among the first methods used for rigidly-jointed frameworks, both with regard to portals and trusses, and elastic theory of suspension bridges. It featured, for example, notably in Müller-Breslau's book *Die neueren Methoden* (1886*b*) which represented a major advance in the literature of theory of structures.

The fear of failure of major bridge trusses, due to the stresses induced by the rigidity of joints (secondary stresses), stimulated research by a number of distinguished German engineers after 1877. In that year, according to Grimm (1908), a prize was offered by the Polytechnikum of Munich for the solution of that problem. The term *Sekundärspannung* (secondary stress) was, it seems, originated by Professor Asimont of that institution, to distinguish between the direct and bending-stresses in an eccentrically loaded column. Asimont formulated the problem with regard to rigidly-jointed trusses and Manderla's solution (1879) gained him the prize, it appears. Before the publication of Manderla's solution in 1880, however, Engesser had published an approximate method. Also, Winkler indicated, in a lecture on the subject (1881), that he had given attention to it for some years past. In 1885 Professor Landsberg contributed a graphical solution which was followed by another analytical solution by Müller-Breslau in 1886 (1886*a*). Another graphical solution appeared, it seems, in 1890, this time from W. Ritter; and then in 1892 a further analytical solution was contributed by Mohr. Engesser published a book on the analytical determination of secondary and additional stresses in 1893. These aspects are considered in more detail in Chapter 11.

Although Müller-Breslau used Castigliano's energy method to analyse a simple (single storey, single bay) rigidly-jointed portal framework, that kind of structure did not, it seems, attract the degree of attention given to secondary effects in trusses until the twentieth century. The appearance of Vierendeel's novel design for open-panel bridge girders in 1897 (according to Salmon, 1938) posed a formidable analytical problem for which approximate methods (for example the use of estimated points of contra-flexure) were used initially.

This review of nineteenth-century structural engineering, with reference to theory of structures, would be incomplete without mention of the problems posed by major suspension bridges. In spite of adverse experience, particularly in Britain, the Americans persevered and in 1855 adopted the suspension principle successfully for a major railway bridge to Roebling's design, over the Niagara Falls. Although, at that time, theory of suspension bridges (Chapter 3) was based on simplifying assumptions to render the problem amenable to statics alone (as, for example, in

Rankine's theory, 1858) and, moreover, the provision of ties from deck to towers to supplement the cables was common, it seems the concept of gravity stiffness was being recognised implicitly, if not explicitly. Thus, Pugsley (1957, 1968) quotes from a letter Roebling sent to the company prior to the building of the Niagara Bridge: 'Weight is a most essential condition, where stiffness is a great object.' Then in 1883 the great Brooklyn Bridge was built to Roebling's design, judged by Pugsley to be a triumph of intuitive engineering. The completion of this bridge almost coincided with the origins of elastic theory to which W. Ritter (1877), Fränkel (1882), Du Bois (1882) and Levy (1886) were early contributors (Chapters 3 and 9), and in which elastic deflexion of the deck, due to live load, determined the uniformly distributed reaction provided by the cables, as dictated by compatibility of displacements. Approximation regarding small deflexion of the cables was involved and substantial improvement in this respect, following the introduction of gravity stiffness by Melan (1888), whereby accurate analysis of bridges with relatively flexible decks (which depended greatly on that source of stiffness), became possible. The modern deflexion theory of suspension bridges is due essentially to Melan and then to Godard (1894).

In conclusion it is interesting to recall Pole's commentary (Jeaffreson, 1864) on iron bridges, with emphasis on British practice. He remarks that the history of iron bridges commenced in the sixteenth century when such structures were proposed in Italy and then that an iron bridge was partly manufactured at Lyons in 1755, but that it was abandoned in favour of timber in the interests of economy. In the event, however, the first iron bridge (of cast iron) was erected in Britain, being completed in 1779. The builder was Abraham Darby and the site was the River Severn at Coalbrookdale, Shropshire. But Pole notes that as early as 1741 a wrought iron-chain footbridge was erected over the River Tees, near Middleton, County Durham. That kind of bridge was later developed by Captain Samuel Brown (for example the Union Bridge of 1819 near Berwick) who used long iron bars instead of ordinary link chains. (In the U.S.A., Finlay built an iron suspension bridge in 1796 at Jacob's Creek, and Séguin's bridge at Ainé was built in 1821.)

Pole notes the impetus to bridge engineering, given by the coming of railways, and the need to discover an alternative to the arch for a variety of circumstances. He suggests that the simple beam ('the earliest form of all') made of iron, afforded the desired economical solution with its possibilities for development. The 'five great properties' claimed for it were:

- 1 rigidity;
- 2 convenience with regard to being level or straight (for example the Britannia Bridge);
- 3 simplicity of abutment conditions (that is, no horizontal thrust);
- 4 the ironwork for a beam is less in weight than for an arch of the same strength and span;
- 5 convenience of erection and less interference with navigation during construction over water courses.

Pole summarises iron bridges as belonging to three classes: the iron arch; the suspension bridge; and the iron girder bridge, with the last of greatest utility in the eight categories:

- 1 solid beams;
- 2 trussed cast iron girders;
- 3 bowstring girders (cast iron, as in the High Level Bridge by R. Stephenson at Newcastle; wrought iron at Windsor, by Brunel);
- 4 simple 'I' girders;
- 5 tubular or hollow plate girders (e.g. Britannia Bridge);
- 6 triangular framed girders (including the Warren girder);
- 7 lattice girders;
- 8 rigid suspension girders (Brunel's Chepstow Bridge and Saltash Bridge).

The remainder of Pole's article is devoted to specific instances of the use of the various kinds of iron bridge and the failure of some of them, notably Stephenson's trussed cast iron girder bridge at Chester in 1847.

Notes

Robison contributed articles on applied mechanics to the *Encyclopaedia Britannica* (1797) and Brewster (1822) refers to them and to Young's version of them in the next edition.

Straub (1952) and Timoshenko (1953) provide details of the foundation of l'Ecole Polytechnique and l'Ecole des Ponts et Chaussées.

Straub (1952) observes that the absence of graphical methods in Navier's *Leçons* is in contrast with 'modern' text-books, an observation of questionable validity even thirty years ago.

According to Dempsey (1864), Wild joined Clark as assistant to Robert Stephenson on the Britannia Bridge.

Rankine's Inaugural Lecture to the University of Glasgow is reproduced in his *Manual of applied mechanics* (1858).

Clapeyron and Lamé designed iron suspension bridges (1826a) which were constructed during the years 1824-6 in St Petersburg and were among the first on the continent of Europe.

The concept of influence lines seems to have arisen (1868) through Winkler's work in relation to elastic arches (Chapter 3).

Mosely and Fairbairn were distinguished as the only British Corresponding Members in Mechanics of the French Academy in 1858, the year of Clapeyron's election in preference to Saint-Venant and three others, to fill the vacancy due to Cauchy's death. In the same year the Academy appointed Clapeyron to a Commission to advise on the Suez Canal project.

Cotterill (1869) brought to the attention of British engineers the use of the funicular polygon for the graphical calculation of bending moments of simple beams. He referred to a preliminary version of Culmann's *Die graphische Statik* (1866) published in Leipzig in 1864 and to Reuleaux (1865).